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Dynamic Stability Measurements from Tunnel Unsteadiness Excited Random Response

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Flow unsteadiness in transonic and supersonic tunnels often causes large scatter and, when excessive, even impedes dynamic stability measurements by conventional oscillatory techniques. A novel technique utilizing the tunnel unsteadiness as primary excitation on flexure-mounted models for dynamic stability measurements in a trisonic blowdown tunnel is presented in this paper. A time series autoregressive modeling technique is used for deriving a digital spectrum of the unsteadiness excited model response and the system damping is evaluated from the half-power bandwidth of the spectrum. A typical record length used for the spectral analysis is 1.5 s. The technique, validated by comparisons with conventional free-oscillation pitch-damping measurements on two models at subsonic and transonic speeds, is well suited for dynamic stability measurements on stable configurations in short duration, intermittent facilities.

Nomenclature

$a_1, \ldots, a_n$	= autoregressive parameters corresponding to model order n
D	= structural damping
d	= reference length, base diameter of model
C_m	= pitching moment coefficient, pitching moment/$q_\infty Sd$
C_{m_q}	= pitching moment coefficient due to pitch velocity $\partial(C_m)/\partial(qd/V_\infty)$
$C_{m_{\dot\alpha}}$	= pitching moment coefficient due to rate of angle of attack change, $\partial(C_m)/\partial(\dot\alpha d/V_\infty)$
I	= structural inertia
M_∞	= freestream Mach number
N	= observation vector length
n	= model order of the autoregressive description
q	= pitching velocity
q_∞	= freestream dynamic pressure
R_{Nd}	= Reynolds number based on d
R_e	= autocorrelation function
r	= residuals
S	= reference area, $\pi d^2/4$
Δt	= sampling period
V_∞	= freestream velocity
V_n	= loss function
α	= angle of attack
ω	= frequency, rad/s
ω_n	= model natural frequency, rad/s
$\Delta\omega$	= half-power bandwidth
ζ	= damping ratio
$\hat\lambda_{AR}$	= estimated autoregressive parameter vector
τ	= time delay
ϕ_{AR}	= autoregressive spectrum

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I. Introduction

WIND-TUNNEL measurement of dynamic stability derivatives are usually based on conventional free- and forced-oscillation methods. These techniques employ spring-mounted models usually constrained to a single or sometimes two and three degree-of-freedom motion, and the derivatives are obtained by analysis of the model response to an initial step displacement or a steady, sinusoidal forcing input. The unavoidable flow fluctuations in the tunnel freestream constitute one of the chief sources of errors adversely affecting the measurement accuracy of these methods. The tunnel unsteadiness, which is random in nature, acts as an additional and unwanted source of excitation on the model, and the response due to this is superimposed on the deterministic response of the model (decaying or constant amplitude oscillations) resulting in a low signal-to-noise ratio and, consequently, large scatter in measurements. The usual procedure to reduce inaccuracies on this account is to conduct longer duration tests with several damping cycles in the free-oscillation method or long data recording in the forced-oscillation method, and the stability derivatives are evaluated by an averaging process. Such procedures, in addition to requiring long test durations, may not be helpful in some cases when the unsteadiness is high and the signal is largely submerged in noise. Levels of unsteadiness beyond which dynamic stability and other dynamic measurements may not be possible are sometimes specified.[1,2]

The concept of using environmental random disturbance as a test input is currently gaining ground in many branches of engineering.[3] This technique, which dispenses with expensive external excitation systems, is becoming increasingly popular in flight-flutter testing where the atmospheric turbulence is utilized as the primary excitation (Refs. 4 and 5 are examples). Application of this technique for wind-tunnel tests has mostly been for subcritical damping measurements of flutter models,[6,7] and the only attempts to adapt this technique for dynamic stability measurements seem to be those of Refs. 8 and 9.

Several techniques are known for the analysis of random response of linear systems such as power spectral density, autocorrelation, random decrement method, etc. The relative merits of these and other techniques are discussed in Ref. 10 and comparative evaluations of some of these methods are also found in Refs. 6, 7, and 11. The random decrement method, developed by H. A. Cole, Jr.,[12] can be adapted for on-line measurements or for digital computation-based, offline analysis. The requirement of long record lengths seems to inhibit a wider application of this method to short duration facilities. For example, it has been estimated[13] that record lengths of 30-150 s are required for subcritical damping measurements on typical wind-tunnel flutter models. Drane and Hutton[11] reach a similar conclusion: "The time needed to gather sufficient data for analysis would preclude use of the method in all but the longest running intermittent tunnels." Spectral and correlation methods depend on time-averaging analog electronic circuitry and often do not possess adequately low bandwidth resolution. Use of digital techniques for spectrum evaluation of a random signal is much more efficient because of the availability of Fast Fourier Transform and modeling techniques. The latter methods, commonly known as system parameter identification techniques, establish a mathematical model by matching a sampled set of values of the response without resorting to time averaging and, therefore, are generally superior to the conventional spectral analysis methods with regard to bandwidth resolution and ability to separate different modes in a multimode system.[10] Since only a few cycles of response are usually needed for modeling, and the number of samples used generally governs the accuracy, system parameter identification techniques can be adapted for short record lengths and hence are best suited for short duration, intermittent facilities such as the high Reynolds number transonic wind tunnels that are being proposed or built at several places. These facilities typically have about 10-s test duration and, though efforts are underway to achieve reduced flow unsteadiness, availability of measurement techniques suitable for "rough" flow typical of ventilated wall test sections would be advantageous.

This paper presents a description of some experimental investigations for dynamic stability measurements using the flow unsteadiness as the primary excitation on flexure-mounted models in a trisonic blowdown tunnel. The mathematical modeling technique used for evaluating the system resonant frequency and damping from the model random response is discussed in detail. The technique is validated by comparisons with conventional free-oscillation measurements at subsonic and transonic speeds.

II. Analytical Background

The method is schematically ilustrated in Fig. 1. The test model, with its flexure spring support system, is assumed to represent a stable linear dynamical system of unknown degrees of freedom excited solely by the tunnel unsteadiness (though damping evaluation for the present tests was based on a single-degree-of-freedom assumption; the mathematical modeling technique presented here holds good for the general case of a multi-degree-of-freedom system). Furthermore, the excitation caused by the tunnel unsteadiness is assumed to be a weakly stationary "white noise" process. A well-known property of asymptotically stable linear systems excited by the stationary white noise process is that the output of the system, in this case the model response, is a stochastic process of rational spectrum which can be described by an autoregressive model.[14,15] The model response is used for a mathematical model of unknown degrees of freedom whose output correctly matches the recorded response in a least-square sense. Time series analysis concepts are used to fit an autoregressive model whose order is determined by analyzing the residuals of the mathematical fit. The mathematical model is used to evaluate the autoregressive spectrum from which the system resonant frequency and half-power bandwidth and hence the system damping are extracted.

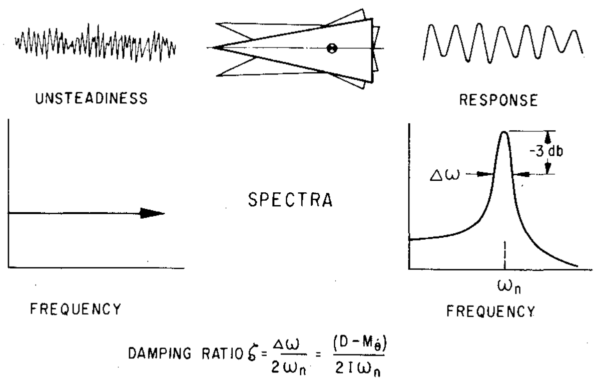

Fig. 1 Schematic illustration of the method.

Autoregressive (AR) Modeling

The output of the system is taken to be that of an nth order linear dynamical system excited by an inaccessible white noise representing the flow unsteadiness and is assumed to be stationary over the period of observation. This continuous stationary random process $\{y(t)\}$ is observed as a discrete process $\{y(k)\}$, $k=[0, 1, 2, \ldots, N+n]\Delta t$ where $N \gg n$ using uniformly spaced samples taken at intervals Δt by a zero-order sampler. The sampling interval is chosen to be compatible with the spectrum of interest.[15] Then,

$$e(k)=A(z^{-1})y(k) \tag{1}$$

represents an nth order autoregressive process, where z^{-1} is the backshift operator representing sampling time; $A(z^{-1})$ is the mathematical model embracing the system parameters, and is represented as $1+a_1z^{-1}+a_2z^{-2}+\ldots a_nz^{-n}$; and $e(k)$ is the independently and identically distributed inaccessible white noise representing the environment and possessing zero mean and a variance of σ^2.

Equation (1) can be rewritten in the prediction form as:

$$y(k)=e(k)-[a_1y(k-1)+a_2y(k-2)+\ldots a_ny(k-n)] \tag{2}$$

The significance of the autoregressive model is that the output of the model is dominantly a function of its previous history over the period $n\Delta t$, with an additive but inaccessible, white noise component and has a predictor capability.

Equation (2) can be rewritten with $e(k)$ replaced by $r(k)$ to represent residuals in the estimation procedure as:

$$y(k)=r(k)-[a_1y(k-1)+a_2y(k-2)+\ldots a_ny(k-n)] \tag{3}$$

If N such observations of $y(k)$ are made consecutively, the set of such observations can be expressed in the vector matrix notation as

$$Y=B\lambda+R \tag{4}$$

where

$$Y=[y(n+1)\ y(n+2)\ \ldots\ y(n+N)]^T$$

$$\lambda=[-a_1 -a_2\ \ldots\ \ldots\ \ldots\ -a_n]^T$$

$$B=\begin{bmatrix} y(n)\ y(n-1) & \ldots & y(1) \\ y(n+1)\ y(n) & \ldots & y(2) \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ y(n+N-1) & \ldots\ldots & y(N) \end{bmatrix}$$

and

$$R = [r(n+1)\ r(n+2)\ \ ..\ \ r(n+N)]^T$$

The parameter vector λ can be evaluated by minimizing the loss function

$$V_n = \tfrac{1}{2} \sum_{k=1}^{N} r^2(k)$$

as

$$\lambda_{AR} = (B^T B)^{-1} B^T Y + (B^T B)^{-1} B^T R \tag{5}$$

With the assumption that residuals represent the inaccesible white noise, the second term in Eq. (5) tends to zero for a large observation vector length N. Consequently, $r(k) = e(k)$ and

$$\hat{\lambda}_{AR} = (B^T B)^{-1} B^T Y \tag{6}$$

Equation (6) is identical to the autoregressive model fit that can be obtained from the Yule-Walker expression.[15] The fit error, which describes the ablity of $\hat{\lambda}_{AR}$ to predict the system response, can be evaluated by the quantity

$$\sum_{k=1}^{N} r^2(k) \Big/ \sum_{k=1}^{N} y^2(k) \tag{7}$$

This is a useful figure of merit.

Model Order Determination

Three methods of determining the model order by analyzing the residual vector are available in the literature.[15,16] A model order is usually guessed and an estimation of $\hat{\lambda}_{AR}$ carried out using Eq. (6). From this estimate, residuals $r(k)$ can be evaluated using Eq. (3).

The first technique is to evaluate the autocorrelation function of the residuals for various time delays. If the residuals are uncorrelated, this tends to an impulse function with correlation function value

$$R_e^2(\tau)/R_e^2(0) \quad \text{less than} \quad 1/\sqrt{N} \quad \text{for} \quad \tau \neq 0$$

where $R_e^2(\tau)$ is the autocorrelation function of $e(k)$, and $k = 0, 1, \ldots, N$ for time lag $\tau = k\Delta t$.

The second technique is the repeated least-squares method wherein the model order statistics Fn_1n_2 are evaluated as

$$F_{n_1 n_2} = \frac{V_{n_1} - V_{n_2}}{V_{n_2}} \; \frac{N - 2n_2}{2(n_2 - n_1)} \qquad N > n_1$$

where n_2 is the new model order and V_{n_1}, V_{n_2} are the loss functions. This is known to possess F distribution.[17] Insignificant change in $F_{n_1 n_2}$ is sought by progressively increasing the model order.

The third technique, due to Akaike,[16] consists of determining an information criterion known as Akaike's Information Criteria (AIC) for a scan of the model order. The AIC is given by

$$\text{AIC}(n_1) = (N + n_1/N - n_1)\,[Q_0 + a_1 Q_1 + \ldots\ a_{n_1} Q_{n_1}] \tag{8}$$

where n_1 is the test model order and

$$Q_j = (1/N) \sum_{k=1}^{N-j} y(k+j)\,y(k),\ j = 0, 1 \ldots, n_1$$

All three methods were applied to determine the appropriate model order of the system under consideration, as subsequently discussed.

Autoregressive Spectrum and System Damping Evaluation

Having evaluated the model order n and the associated parameter vector $\hat{\lambda}_{AR}$, the autoregressive spectrum associated with the observation signal can be evaluated by frequency domain transformation as:[15]

$$\phi_{AR}(\omega) = \Delta t \sigma^2 \Big/ \Big[1 + \sum_{k=1}^{n} a_k \exp(ik\omega\Delta t) \Big]$$

where the variance $\sigma^2 = E(e^2)$ and $i = \sqrt{-1}$.

From the autoregressive spectrum, the various modes of the system and the associated natural frequency and bandwidth can be identified.

The autoregressive modeled spectral estimates so obtained are unbiased, and if the number of N samples is sufficiently large, the finite-order, autoregressive model can be expected to estimate the spectrum of the system arbitrarily close.[18]

For the present system, the model-spring combination is assumed to be a single-degree-of-freedom system with constant parameters excited by tunnel unsteadiness. The governing differential equation for pitching oscillations of the model θ is:

$$I \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + K\theta = P(t) + M$$

where I, D, and K are the structural inertia, damping, and stiffness, respectively. For small displacements, the pitching moment M is written as:

$$M = M_\theta \theta + M_{\dot\theta} \dot\theta$$

where M_θ and $M_{\dot\theta}$ are the aerodynamic stiffness and damping derivatives.

When the forcing function $P(t)$ is a white noise process, the bandwidth at "half-power" point on the response spectrum can be shown to be:[19]

$$\Delta\omega = 2\omega_n \zeta$$

where ω_n is the resonant frequency and the damping ratio ζ equals $(D - M_{\dot\theta})/2I\omega_n$. Hence, the system damping is obtained as

$$(D - M_{\dot\theta}) = \Delta\omega I$$

The structural terms I and D are measured in a conventional wind-off, free-oscillation test, and the "half-power" bandwidth is obtained from the autoregressive spectrum.

III. Experiments

Wind Tunnel

Tests were conducted in the 1-ft blowdown tunnel at NAL. This is an atmospheric discharge tunnel operated from compressed air stored at a maximum pressure of 150 psig. A vertical axis rotating plug valve controls the tunnel stagnation pressure. A radial splitter flow-spreading system installed in the wide-angle diffuser preceding the settling chamber helps to reduce the flow unsteadiness.[20] The tunnel is operated at a stagnation pressure of 26 psia at subsonic and transonic speeds. The 12 x 15 in. transonic test section has an 8% slotted roof and floor and solid sidewalls.

Apparatus

The basic apparatus used for the tests was a wall-mounted oscillation rig shown in Fig. 2. An interchangeable, three-strip, crossed-flexure pivot anchored on the tunnel sidewall provides the elastic suspension for the model. A reflection plate enables the model to be supported outside the tunnel wall boundary layer. A mechanically actuated spring-loaded

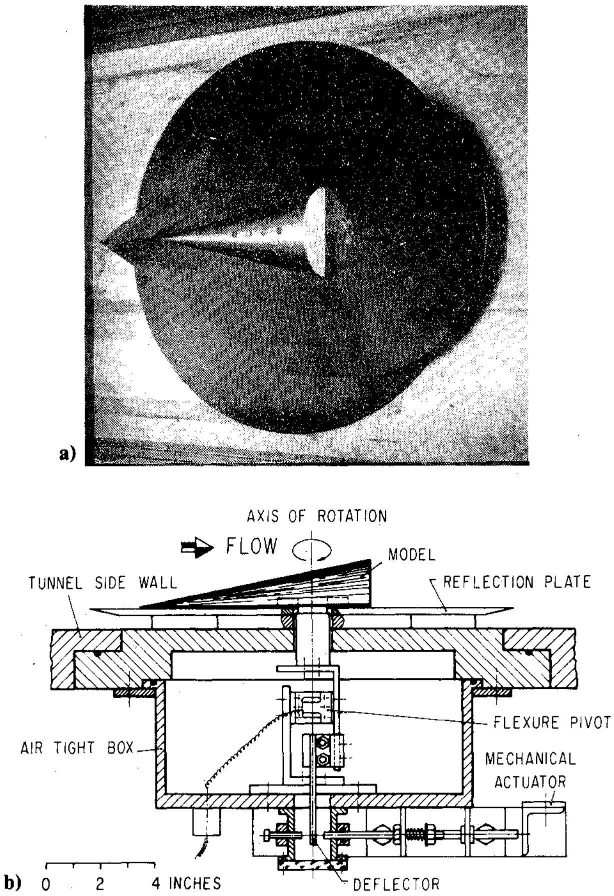

Fig. 2 Oscillation rig: a) installation photograph; b) plan view.

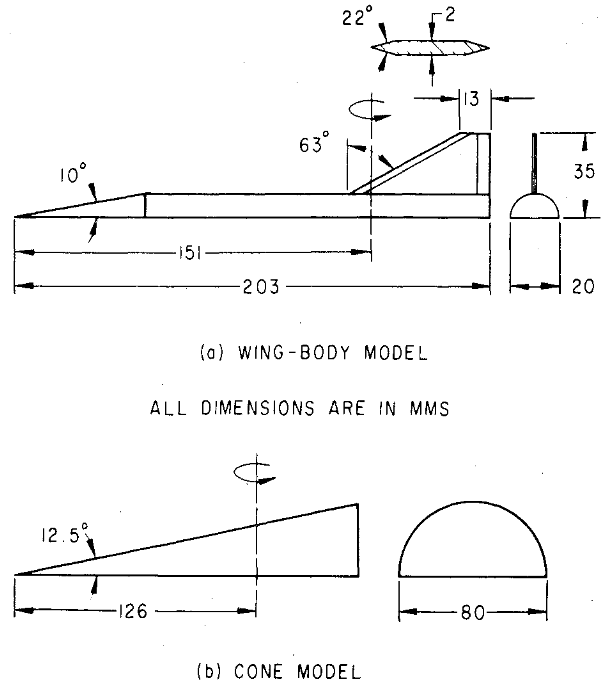

Fig. 3 Details of models.

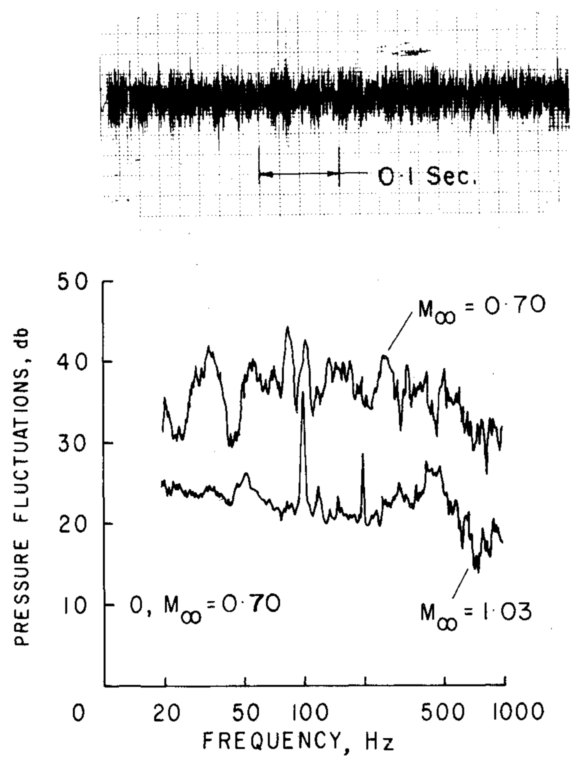

Fig. 4 Typical pressure fluctuations.

device triggers the decaying oscillations from a preset initial amplitude. Relatively stiff flexure pivots were used to reduce the unsteadiness-excited displacement amplitude of the model to low values to enable meaningful free-oscillation measurements by a conventional log decrement method.

A strain-gage bridge mounted on the center strip of the flexure pivot provides the model response signal which is amplified and recorded simultaneously on a FM magnetic tape recorder and an oscillographic recorder. In order to obtain a qualitative nature of the tunnel unsteadiness, a static pressure fluctuation signal from a 25 psia ALINCO pressure transducer mounted in a suitable housing on the sidewall was recorded on the tape recorder. The pressure fluctuations were analyzed with a B & K wave analyzer and level recorder. The autoregressive modeling and the spectrum were evaluated by a program developed on an IBM 360 computer. Typical computation time was 2 min.

Models and Wind-Tunnel Tests

Two half-models (Fig. 3) were used for the tests. The wing-body model had a fineness ratio of 10 and a 63-deg cropped-delta wing with a symmetrical double-wedge section of constant thickness. This was tested with the axis of oscillation at 75% of its length from the nose. The other model was a 25-deg included angle cone with the axis of oscillation at 70% of the length from the apex. The gap between the model plane of symmetry and the reflection plate was 0.75 mm for both models.

In a typical blowdown of about a 30-s duration, the model response to tunnel unsteadiness was recorded for the first 8 s after flow establishment in the tunnel. In the subsequent 22-s test duration, conventional decaying oscillation cycles were triggered successively ten times. Wind-off, free-oscillation records were obtained before and after each blowdown. The initial amplitude for all free-oscillation tests was 1.5 deg. The maximum amplitude of the model random response to flow unsteadiness varied with Mach number and was between 0.1 and 0.2 deg.

IV. Results and Discussion

Typical static pressure fluctuations spectra presented in Fig. 4 show the existence of a wide band frequency in the tunnel flow and, though wavy, particularly at $M_\infty = 0.7$, they are devoid of many sharp peaks near frequencies of interest (up to about 100 Hz). It is pertinent to note that, at best, the static pressure fluctuations reflect only a qualitative picture of the

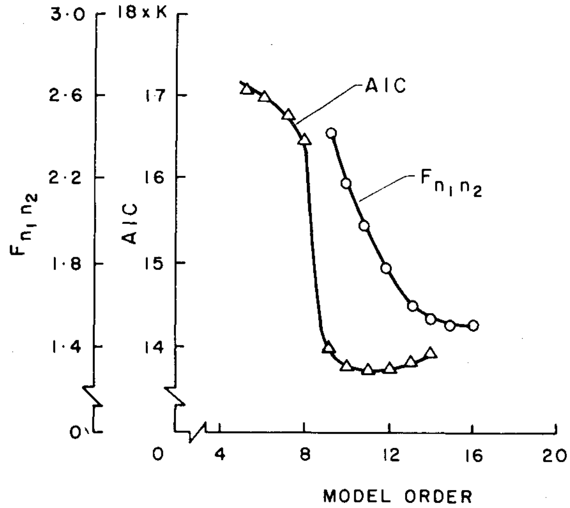

Fig. 5 Model order determination.

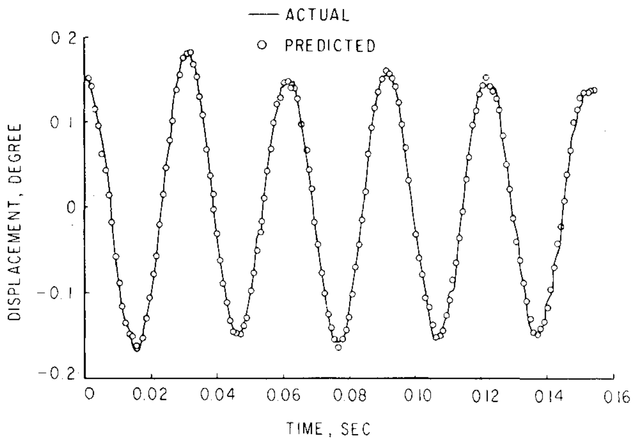

Fig. 6 Actual and predicted time histories of model response.

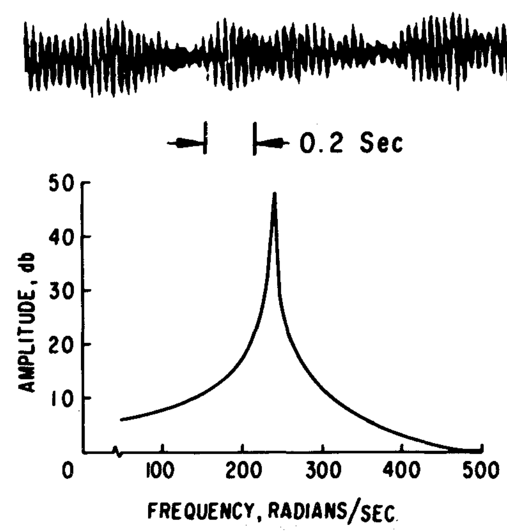

Fig. 7 Typical random response of the model.

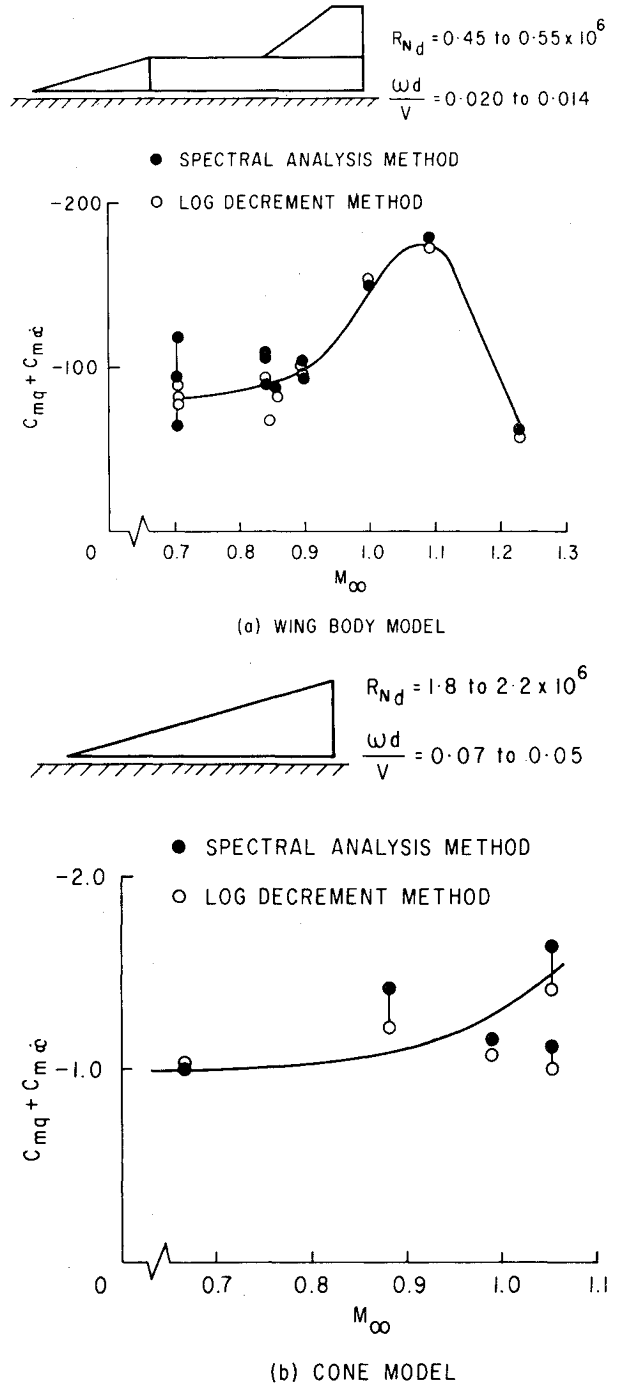

Fig. 8 Comparison of pitch-damping derivatives from spectral analysis and free-oscillation methods.

tunnel unsteadiness, and the excitation on the model depends, in general, on the nature and relative magnitudes of fluctuations in freestream static and total pressures, velocity, and temperature.

The autoregressive model order was determined using the repeated least-squares method and the AIC for a scan starting from 5 to 16. Figure 5 shows the fit error [Eq. (7)] and the AIC [Eq. (8)] as a function of the model order. The autocorrelation functions of the residuals for the various time delays were evaluated and it was found to be an impulse function with about 6% of the correlations beyond the $1/\sqrt{N}$ band limits, as discussed earlier. The observed insignificant change in the fit error and the minima of AIC (Fig. 5), which indicates an unambiguous model order, suggest the best choice of model order as 13. The fit error of the autoregressive model description was less than 1.5%.[21]

Typically, the observation vector length N, used for obtaining the autoregressive spectrum of the model response, was 1500 sampled data points corresponding to about 50 cycles of time history (or 1.5 s), and is far greater than the model order of the autoregressive description. The analysis carried out in some cases for 1000, 1500, 2000, and 2500 sampled data points did not show any observable difference.

The actual and predicted response time histories and the corresponding spectrum in a typical case are shown in Figs. 6 and 7, respectively. The excellent agreement between two time histories can be noted. The measured bandwidth varied between 0.1 and 0.4 Hz, corresponding to damping ratios between 0.003 and 0.01. In some cases, the response analysis was carried out for three different samples from the same test and the extracted damping ratios agreed within 3%. This fact, combined with a low error covariance (less than 1.5%), suggests good confidence in the results.

The measured pitch-damping derivatives from the spectral analysis and conventional free-oscillation methods are shown in Fig. 8. The average value of ten damping cycles is presented as one data point for the free-oscillation tests. A typical standard deviation of these values was 20%. Results from the two measurement methods are seen to be in excellent agreement for the two models in the test range of Mach numbers. The good agreement shown, despite some discrepancies between the nature of assumed (flat) and actual (wavy) spectra of tunnel unsteadiness, is noteworthy. This indicates that the nature of unsteadiness spectrum a few octaves beyond the model natural frequency may not affect the results. It is believed that this would, in general, be true for the usual high-speed tunnel dynamic stability testing systems which have very low damping ratios (or high Q-factors)[22] and, consequently, the model response to all frequencies, but a few octaves around the system resonant frequency, is negligible, Unsteadiness spectra in several existing tunnels[23] indicate that the characteristic spectrum at transonic speeds is approximately flat in the region of interest for dynamic stability measurements and advantage can be taken of the existing tunnel unsteadiness for conducting relatively more accurate (when flow unsteadiness is high) and less expensive dynamic stability tests, as compared to conventional techniques. However, large waviness in unsteadiness spectrum around the model natural frequency may affect the accuracy.

V. Conclusions

The technique of utilizing the flow unsteadiness as the primary excitation for dynamic stability measurements in a trisonic wind tunnel has been demonstrated. An autoregressive modeling technique enables the use of short record lengths for deriving a digital specrum of model random response. The method can, in general, be used for dynamic stability measurement of stable configurations, and would be particularly advantageous when large tunnel unsteadiness and the consequent poor signal-to-noise ratio renders the conventional free- and forced-oscillation techniques unsatisfactory or inapplicable. Other advantages of the method are: 1) no external excitation equipment is needed; 2) short record lengths are adequate for accurate measurements; and 3) bandwidth resolution is not a limitation. The method is well suited for dynamic stability measurements in short-duration, transonic wind tunnels. Difficulties in trimming large static moments when the axis of oscillation is located far from the center of pressure or extensive flow separation on the model may limit the application of the method to moderate incidences.

Acknowledgments

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